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Random walk with memory enhancement and decay

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A model of random walk with memory enhancement and decay was presented on the basis of the characteristics of the biological intelligent walks. In this model, the movement of the walker is determined by the difference between the remaining information at the jumping-out site and jumping-in site. The amount of the memory information $s_i(t)$ at a site i is enhanced with the increment of visiting times to that site, and decays with time t by the rate $e^{-\beta t}$, where β is the memory decay exponent. When $\beta=0$, there exists a transition from Brownian motion (BM) to the compact growth of walking trajectory with the density of information energy u increasing. But for $\beta>0$, this transition does not appear and the walk with memory enhancement and decay can be considered as the BM of the mass center of the cluster composed of remembered sites in the late stage.

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I. INTRODUCTION

During the last two decades, a lot of complex phenomena including fractal growth and random walks have attracted considerable interest. Since critical phenomena made us appreciate the presence of power laws in nature, random walks became a paradigm of various models involving stochastic motion [1–4]. The traditional random walks, such as Brownian motion (BM) have been studied in great detail [1–3]. In recent years, much attention has been paid to the random walks with interaction including self-avoiding walk, random walk on percolation, active walk, and so on [1–7]. Self-avoiding walk describes the statistic behavior of polymer in solution [5]. Random walks on fractals present the characters of abnormal transport properties for fractal systems [4,6]. Active walk is applicable to the study of river formation, ant swarms, and so forth [1,7].

In addition, there has been an increasing interest in the research of biological motions, such as migration of fish, flocks of flying birds, and animal aggregations [8–10]. Vicsek *et al.* introduced a model to describe the self-ordered motion of biological individuals, in which the velocity of a given particle is related to those of the neighboring particles [10]. The model gives the picture of cooperation motion, but the relations between the velocities of particles have some degree of artificialness. Very recently, a model of self-attracting walk (SATW) was introduced [11–13]. In the model, a random walker jumps to the nearest neighbor sites with jumping probability $p\propto\exp(nu)$, where $n=1$ for already visited sites and $n=0$ for unvisited sites. u stands for the attractive interaction. For $u>0$, the walk is attracted to its own trajectory. Since n only takes two values: 0 or 1, the model is too simple to describe the variation of memory with time [14]. In a previous work, we presented the "true" SATW model involved in the enhancement of memory with the increase of visited times [15]. Besides memory enhancement, for some biological intelligent walks, such as the walks of insects, the memory also decreases with the passage of time [16]. Based on the bionics, we will propose a model of intelligent random walk, which includes the main characters of the biological walks, e.g., the walk of ants. Considering the number of times a site is visited as the variation of the environmental state made by the walker, the motion of the walker is restricted by the visiting number, which is the effect of environment on the walker. In this sense, the present model can be extended to a general form: *the walk changes the environment, and the varied environment affects the walk in return*. Furthermore, *the effect of the varied environment decays with the passage of time*.

In this paper, we present a model of random walk with memory enhancement and decay based on the characteristics of the biological intelligent walks. The results will be helpful to understand the behavior of the walks in complex systems interacting with environment, such as the behaviors of insects, animals, and collective motion of robots.

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II. MODEL AND METHOD

The Monte Carlo (MC) method has been used in the simulations. The walker moves on the sites of a square lattice from a certain site i to its neighboring site j with the jumping probability

$$p_{ij}\propto\exp(\Delta U_{ij}/k_BT), \tag{1}$$

where ΔU_{ij} is the energy difference between jumping-out site i and jumping-in site j. k_B is the Boltzmann constant. For the case of constant temperature, k_BT is set to an unity. In the present model,

$$\Delta U_{ij}=\Delta s_{ij}u, \tag{2}$$

where Δs_{ij} is the difference between the amount of information at jumping-out site i and that at jumping-in site j, i.e., $\Delta s_{ij}=s_j-s_i$. u is the density of information energy, which is the variation of energy generated by unit information. $u>0$ indicates that the walker tends towards the sites with strong information. Now, the information comes from the memory of visited times by the walker. We express the visiting times at MC time m and suppose the information declines with time by the rate $e^{-\beta t}$. Thus, for the site i, the remaining information $s_i(t)$ at MC time t is the accumulation of the remembered visiting times, i.e.,

$$s_i(t)=\sum_m n_i(m)e^{-\beta(t-m)}, \tag{3}$$

where β is the memory decay exponent and $\beta \geqslant 0$. $n_i(m)$ is taken as 1 (if site i was visited at time m) or 0 (if site i was not visited at time m). Therefore, the probability p_{ij}, with which the walker jumps from site i into its neighboring site j, can be written as

$$p_{ij} \propto \exp[(s_j - s_i)u]. \tag{4}$$

In addition, at time t, the remembered sites are the sites with nonzero remaining information at this moment, i.e.,

$$s(t)>0. \tag{5}$$

In the simulations, we took 10^{-7} as the nonzero criterion. In reality, when the amount of information exceeds a certain value, the effect does not increase anymore [14–16]. Therefore, we introduce a saturated-information amount s_m to express the superior limit. The restriction on s can be described as $s(t) \leqslant s_m$. When $\beta=0$, i.e., the information never declines, the present model degenerates to a ''true'' SATW one [15]. As $\beta \to \infty$, it reduces to the pure BM.

The structural characteristics of the walking trajectory can be described by the mean-square end-to-end distance $\langle R^2(t) \rangle$ and the average number of visited sites $\langle S(t) \rangle$. It is expected that there exist scaling relations between these two quantities and time t as

$$\langle R^2(t) \rangle \propto t^{2\nu}, \tag{6a}$$

and

$$\langle S(t) \rangle \propto t^{k}, \tag{6b}$$

where ν and k are the scaling exponents.

III. RESULTS AND DISCUSSIONS

The numerical simulations in two-dimensional (2D) space are performed for variant memory decay exponent β and densities of information energy u.

In the case $\beta=0$, there exists a transition from random walk behavior to SATW one. The transition point u_c was named for the critial density of the information energy and it decreases with the increase of the saturated-information amount s_m. We have that $u_c \simeq 0.88$ for $s_m=1$ and $u_c \simeq 0.1$ for $s_m=13$ [15]. Figure 1 plots the log-log relation of $\langle R^2(t) \rangle$ and $\langle S(t) \rangle$ to time step t with variant u for a very small value of β ($\beta=10^{-4}$). It is found that, there exists an interesting phenomenon that all of these lines are parallel to that of BM in the long-time stage, and the scaling exponents $\nu \sim 0.5$ and $k \sim 0.9$ are the same as those of BM [17]. In the case of small u, the result is easy to understand because the effect of information energy on the walk is little and the walk is dominated by the thermal fluctuation. But for the case $u > u_c$, the result is unconformable with our expectation. By

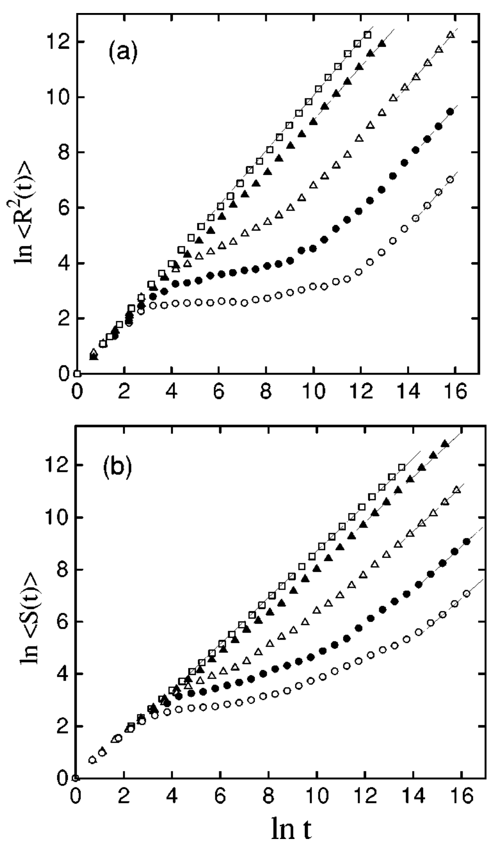

FIG. 1. The log-log plots of mean-square end-to-end distance $\langle R^2(t) \rangle$ (a) and average number of visited sites $\langle S(t) \rangle$ (b) to time step t for the random walk with memory enhancement and decay. The saturated information amount $s_m=13$ and decay exponent $\beta=10^{-4}$. For the ''true'' SATW with $s_m=13$, the critical attractive energy $u_c \simeq 0.1$ [15]. From top to bottom, the density of information energy u is taken as 0, 0.1, 0.2, 0.3, and 0.4, respectively.

the inference, in the case of very small β, the scaling exponents ν and k should be close to those in the case $\beta=0$, and the present results should take the values of the ''true'' SATW, $\nu \simeq 1/3$ and $k \simeq 2/3$ [15]. But our simulations show, when $u > u_c$, the walk with $\beta>0$ belongs to a different universal class from that for the walk with $\beta=0$. When $\beta=0$, there is a transition from BM ($\nu \simeq 0.5$ and $k \simeq 0.9$ [17]) to the compact growth of walking trajectory ($\nu \simeq 1/3$ and $k \simeq 2/3$) with u increasing [12,15]. But for $\beta>0$, the transition exists no longer and we get $\nu \simeq 0.5$ and $k \simeq 0.9$ for all values of u. The changes of s_m and β affect the initial behavior rather than the asymptotic behavior for the curves in Fig. 1. With the increase of s_m or the decrease of β, the initial stage will become long except for the case $u=0$. However, the asymptotic behavior remains the same as that of BM. In the following, we make a thorough inquiry to the specious inference on the case $u > u_c$.

Figure 2 plots the trajectories of the walks with large density of information energy u in two cases, $\beta=0$ and $\beta>0$. Figure 2(a) corresponds to the case $\beta=0$. The cluster consisting of visited sites appears pie shaped and its dimension is the same as that of the space. Moreover, the cluster grows by its edge spreading out, just like the growth of the Eden

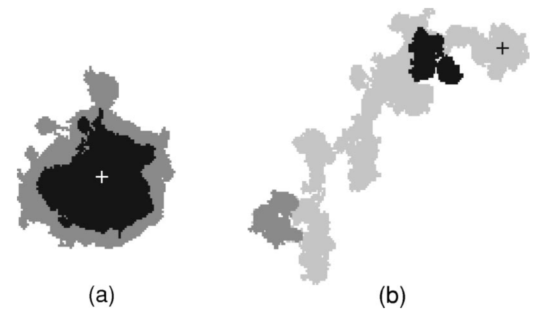

FIG. 2. The clusters consisting of the visited sites in the case of the density of information energy $u=0.35$ and saturated information amount $s_m=13$. (a) The random walk with memory enhancement but without memory decay ($\beta=0$), $t=6\times10^6$ (in black) and $t=16\times10^6$ MC steps (in dark gray). (b) The random walk with memory enhancement and decay. The decay exponent $\beta=10^{-4}$. All visited sites at $t=16\times10^6$ are denoted by light gray points. The remembered sites, which have been visited and process nonzero remaining information, are marked by black points (at $t=6\times10^6$) and dark gray points (at $t=16\times10^6$), respectively. The starting points are indicated by white (a) and black (b) crosses. For the "true" SATW with $s_m=13$, the critical attractive energy $u_c\simeq0.1$ [15].

cluster [18,19]. Figure 2(b) shows two clusters consisting of the sites with $s(t)>0$ at $t=6\times10^6$ and 16×10^6 MC steps, respectively, for $\beta=10^{-4}$. These two clusters indicate the remembered sites at these two certian moments. They are rather compact. Their dimensions calculated by the box-counting method are about 1.9 [2,19]. Their morphologies are variable, but the sizes appear to be about the same. The total number of visited sites looks to be the traces left by the movement of the variable-form cluster composed of the sites with $s(t)>0$. To check the validity of the visual observation from Fig. 2(b), we have calculated the mean number $\langle S_c(t)\rangle$ of the sites with $s(t)>0$. The results are shown in Fig. 3. It can be seen that in the initial stage, the visited sites are still

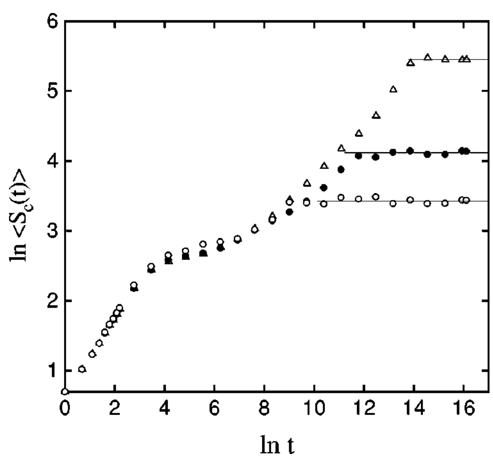

FIG. 3. The log-log plots of the mean number $\langle S_c(t)\rangle$ of the remembered sites to time step t for the random walk with memory enhancement and decay. $u=0.4$ and $s_m=13$. For the "true" SATW with $s_m=13$, the critical attractive energy $u_c\simeq0.1$ [15]. From top to bottom, the decay exponent β is taken as 10^{-4}, 10^{-3}, and 10^{-2}, respectively.

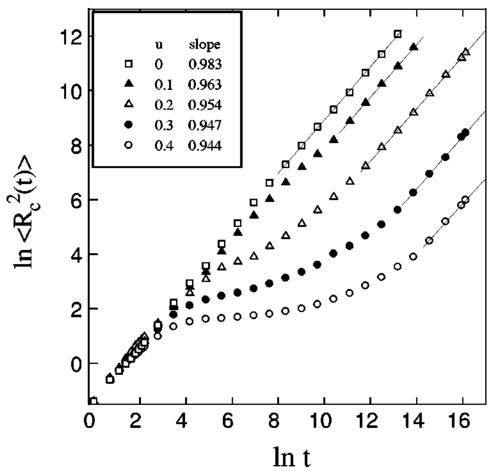

FIG. 4. The log-log plots of the mean-square end-to-end distance $\langle R_c^2(t)\rangle$ of the mass center of the cluster consisting of the remembered sites versus time step t for the random walk with memory enhancement and decay. The decay exponent $\beta=10^{-4}$. The saturated information amount $s_m=13$. From top to bottom, $u=0$, 0.1, 0.2, 0.3, and 0.4, respectively. For the "true" SATW with $s_m=13$, the critical attractive energy $u_c\simeq0.1$ [15]. The given values for the slopes are for a certain time t and these values will approach $\nu_c=1/2$ for increasing t.

remembered and newly visited sites are accumulated in the memory. So $\langle S_c(t)\rangle$ increases with the passage of time. The behavior of $\langle S_c(t)\rangle$ is the same as that of $\langle S(t)\rangle$ in the early stage in Fig. 1(b). But after a certain time t_c, a part of visited sites is forgotten and newly visited sites supplement to remembered sites. It maintains the number of remembered sites close to an equilibrium value, i.e., $\langle S_c(t)\rangle$ remains a constant in the later stage. Based on Eqs. (3) and (5), and considering that the recent visits make much contribution to $\langle S_c(t)\rangle$, we have $s_m e^{-\beta t_c}\lesssim10^{-7}$ (the nonzero criterion) approximately. By solving the inequality, we can get $t_c \gtrsim(1/\beta)\ln(10^7 s_m)$. Corresponding to $\beta=10^{-2}$, 10^{-3}, and 10^{-4}, the expected values are $\ln(t_c)\gtrsim$ 8, 10, and 12, respectively, for the case $s_m=13$. They are consistent with the turning points of the $\ln\langle S_c(t)\rangle$-$\ln t$ curves in Fig. 3.

Furthermore, we have computed the mean-square end-to-end displacement $\langle R_c^2(t)\rangle$ of the mass center of the cluster composed of the sites with $s(t)>0$. The results are shown in Fig. 4. It can be seen that, for $u>u_c$, the scaling exponent ν_c of $\langle R_c^2(t)\rangle$ to t is still close to that of BM in the late stage, i.e., $\nu_c\sim0.5$. Comparing Fig. 4 and Fig. 1(a), we can find that they are rather similar to each other, but different in details. The most important characteristic is that the long-time scaling exponent ν_c of $\langle R_c(t)\rangle$ is identical to ν of $\langle R^2(t)\rangle$.

To get a deep understanding of the difference between the behavior of $\langle R_c^2(t)\rangle$ and $\langle R^2(t)\rangle$, the relationship between $\langle R_c^2(t)\rangle$ and $\langle R^2(t)\rangle$ needs to be set up. When $u>u_c$, the cluster consisting of the remembered sites is rather compact and the dimension d is close to the space dimension. In the vector diagram, $\mathbf{R}_c$ is the vector from the mass center of the cluster to the origin point, $\mathbf{r}$ is the vector from the mass center to the walker, and θ is the angle between $\mathbf{R}_c$ and $\mathbf{r}$.

Then the mean-square end-to-end displacement $R^2(t)$ of the walker can be written as

$$R^2 = R_c^2 + r^2 + 2R_c r \cos(\theta). \tag{7}$$

For $u > u_c$, it takes a long time for the walker to jump to an unremembered site. Before doing this, the walker diffuses around on the remembered sites [with $s(t) > 0$] [12,15]. Considering the mean effect for the cluster, Eq. (7) can be converted into

$$\langle R^2 \rangle = \frac{\int_0^b \int_0^{2\pi} [R_c^2 + r^2 + 2R_c r \cos(\theta)] r\, d\theta dr}{\int_0^b \int_0^{2\pi} r\, d\theta dr}, \tag{8}$$

where b stands for the equilibrium radius of the cluster consisting of the remembered sites. After the integration, Eq. (8) becomes

$$\langle R^2 \rangle = \langle R_c^2 \rangle + b^2/2. \tag{9}$$

Now we make a comparison between $\langle R^2 \rangle$ [in Fig. 1(a)] and $\langle R_c^2 \rangle$ (in Fig. 4) for $u > u_c$ using Eq. (9). In the case $\beta = 0$, the mass center of the cluster can be considered to be immobile in the late stage [see Fig. 2(a)], then $\langle R_c^2 \rangle \sim 0$. $\langle R^2 \rangle$ increases with time by the exponent $\nu = 1/2$ (in the early stage) and 1/3 (in the late stage) for $u > u_c$ [15], as mentioned above. In the case $\beta > 0$, since the memory decay of the information generated by early visits, the cluster tends to grow up towards a certain direction in which the recently visited sites are crowded together. Thus, the mass center is movable, as shown in Fig. 2(b). With the passage of time going, the size b of the cluster composed of the remembered sites increases. When $t > t_c$, b stops increasing and approaches a fixed value [see Fig. 3]. According to Eq. (9), when $t < t_c$, the increment of $\langle R_c^2 \rangle$ with time t is less than that of $\langle R^2 \rangle$. This is the reason why $\ln\langle R_c^2 \rangle$ in Fig. 4 is less than $\ln\langle R^2 \rangle$ for $t < t_c$. When $t > t_c$, b approximates to a constant and $\langle R_c^2 \rangle$ continues to increase with time. So Eq. (9) becomes $\langle R^2 \rangle \approx \langle R_c^2 \rangle$ in the late stage. It comes to the results that, in the late stage, the movement of the cluster consisting of remembered sites can be regarded as that of the mass center of the cluster, i.e., the BM of a mass point. Therefore, the exponent $\nu = \nu_c \simeq 0.5$. It is just the solution about the specious interference mentioned at end of the first paragraph in the present section.

The other exponent k of the walk can also be deduced by a simple scaling analysis. The scaling relation of mean number $\langle S(t) \rangle$ of visited sites to root-mean-square displacement $\langle R(t) \rangle$ is given by [11]

$$\langle S \rangle \propto \langle R \rangle^{d_f}, \tag{10}$$

with $\langle R \rangle = \langle R^2 \rangle^{1/2}$, where d_f is the fractal dimension of the cluster consisting of the visited sites. Comparing Eqs. (6) and (10), we get

$$k = \nu d_f. \tag{11}$$

Using the box-counting method [2,19], we obtain that $d_f \simeq 1.8$ in the late stage. Upon the substitution of d_f and ν in Eq. (11), we get that $k \simeq 0.9$, which is the scaling exponent value of BM [17].

In summary, the behavior of the random walk with memory enhancement and decay can be described as follows. In the initial period, the walker starts to move and leaves information on the visited sites. The information amount $s(t)$ increases with the number of visits and decreases with the passage of time. The cluster consisting of the remembered sites [with $s(t) > 0$] grows gradually. When time t exceeds a certain value t_c, the area of this cluster remains relatively stable rather than growing continuously. The reason is that, some early visited sites are forgotten and some newly visited sites supplement. So the number of remembered sites reaches a stable value. Thus, the cluster consisting of the sites with $s(t) > 0$ maintains a certain size and moves randomly in the space. In substance, this movement of the cluster results from the increase and decrease of $s(t)$ at the edge sites of the cluster. Therefore, the larger the decay exponent β is, the smaller the size of the cluster is and the shorter the time t_c is. When $\beta = 0$, the present model degenerates to the "true" self-attracting walk model, and when $\beta > 0$, it is in the universality class of random walk.

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